7.4

Take It to the Max . . . or Min

Linear Programming

LEARNING GOALS

In this lesson, you will:

- Write systems of inequalities with more than two inequalities.
- Determine constraints from a problem situation.
- Graph systems of linear inequalities and determine the solution set.
- Identify the maximum and minimum values of a linear expression.

KEY TERM

- linear programming

Have you ever wondered why some products cost what they do? How does a company determine how much to charge for an item? The formula for determining the cost of an item takes into account many different factors. Some things to consider are the cost of materials to make the product, the cost of labor to make the product, the cost of transporting the product, and of course, the hope of making a profit on the product so that there is more money for new products. However, sometimes even these factors may not be the answer to the perfect cost versus profit. Companies also have to pay attention to how other companies set the price for similar products, whether this item is a big seller, and how much consumers are willing to pay for the product. It takes a lot of information and analysis to determine how much to charge for a product.

Do you think there are ever times when a company sells an item for less than it is worth? When might this happen? Do you think this is a good business strategy?

PROBLEM 1 Tuning In

A company, TVs4U, makes and sells two different television models: the HD Big View and the MegaTeleBox.

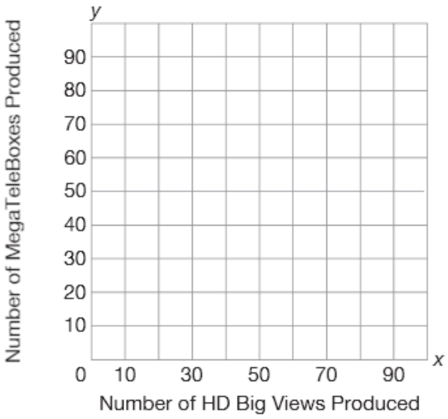
- The HD Big View takes 2 person-hours to make and the MegaTeleBox takes 3 person-hours to make.
- TVs4U has 24 employees, each working 8 hours a day, which is equivalent to 192 person-hours per day.
- TVs4U's total manufacturing capacity is 72 televisions per day.
- TVs4U cannot make a negative number of televisions.

1. Define variables to represent the number of each model television produced.

Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities.

2. Write a system of inequalities to represent the constraints of this problem situation.

3. Graph the system of inequalities on the coordinate plane shown. Shade the region that represents the solution set.



Many companies and businesses are interested in determining when they are maximizing or minimizing their profit or costs. The maximum and minimum values of a system of inequalities occur at a vertex of the region defined by the system.

4. Label all intersection points of the boundary lines.

To determine the maximum and minimum values, you must substitute the coordinates of each point into a given function.

Let's say TVs4U sells the HD Big View for $175 and sells the MegaTeleBox for $205. They want to determine how many of each television they should make and sell to maximize their profits.

Write the function for the given problem situation.	$P(b, m) = 175b + 205m$
Insert the coordinates of each intersection point of the system.	$P(0, 0) = 175(0) + 205(0) = 0$
	$P(0, 64) = 175(0) + 205(64) = 13{,}120$
	$P(24, 48) = 175(24) + 205(48) = 14{,}040$
	$P(72, 0) = 175(72) + 205(0) = 12{,}600$

The maximum profit is represented by the number of televisions made and sold that results in the greatest number.

5. How many of each television should TVs4U produce to earn the maximum profit? Explain your reasoning.

6. TVs4U is trying to determine the price of each model of television. For each set of prices, determine how many of each model should be made to maximize the profit. Then determine the maximum profit. Assume all televisions that are made are sold.

a. The price of the HD Big View is $250 and the price of the MegaTeleBox is $300.

b. The price of the HD Big View is $250 and the price of the MegaTeleBox is $375.

c. TVs4U's boss, Mr. Corazon, sends out a memo with his ideas on maximizing the company's profit.

Mr. Corazon

Obviously, we will make the most money by only making and selling the most expensive television model. Therefore, we should focus on producing and selling 72 MegaTeleBoxes each day for $375 a piece.

Explain why Mr. Corazon's idea is incorrect.

PROBLEM 2 Cashing in on Cell Phones

The cell phone company, Speed of Sound (SOS), produces two types of cell phones. The SOS Smartcall has advanced download speeds and capability which the SOS Basic does not. The assembly lines can produce at most a total of 180 cell phones each day and the company always has at least 40 of each type of cell phone produced and ready for shipping. One SOS Smartcall requires 3 person-hours and $75 of materials to produce. One SOS Basic requires 4 person-hours and $60 of materials to produce. The company has 640 person-hours of labor available daily. The company has budgeted $12,900 for the cost of materials each day.

1. Define your variables and identify the constraints as a system of linear inequalities.

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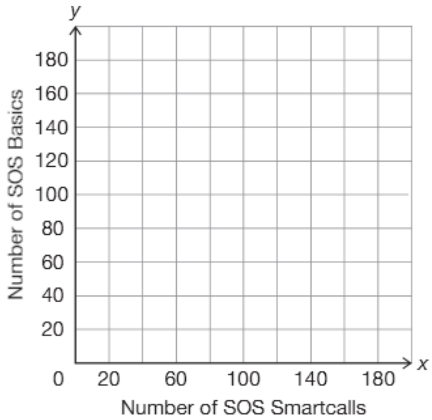
2. Graph the solution set for the system of linear inequalities on the coordinate plane shown. Label all intersection points of the boundary lines.



3. The profit from the Smartcall is $30 and the profit from the Basic is $35.

   a. Write a function to represent the total profit.

   b. Paige states that this problem is unrealistic because no one would ever sell a really good cell phone for only $30. Is Paige's statement correct? Why or why not?

4. How many of each type of cell phone should the company produce and sell to maximize its profit? Determine the maximum profit.

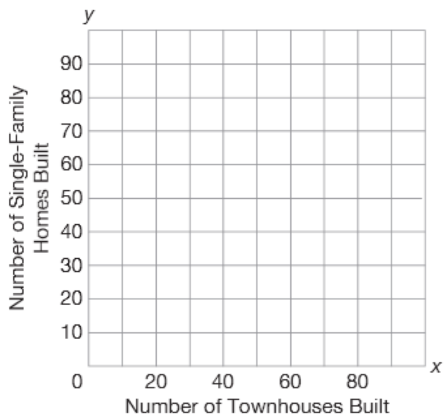
5. A competitor has reduced the price of its advanced capability cell phone. In order to compete, SOS will have to decrease its profit on the Smartcall to $25. How will this affect the number of cell phones SOS needs to produce and sell in order to maximize its profit?

PROBLEM 3 Planning a Housing Development

A building developer is planning a new housing development. He plans to build two types of houses: townhouses and single-family homes. The developer bought a plot of land to build on which already has 20 townhouses and 10 single-family homes built on it. The plot of land has room for the developer to build 100 more homes. It takes the workers 2 months to build a townhouse and 3 months to build a single family home. The developer wants this development complete in 20 years.

1. Define your variables and identify the constraints as a system of linear equations.

2. Graph the solution set for the system of linear inequalities on the coordinate plane shown. Label all intersection points of the boundary lines.



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3. It costs the developer $300,000 to build each townhouse and $450,000 to build each single-family home. Just as the project begins the developer realizes the housing market is not good. He is worried he will not be able to sell all his homes and wants to save his money, but he still needs to build the homes. How many of each type of home should he build if he wants to minimize his costs while still completing the development?

4. The developer sells the houses himself. He sells each townhouse for $325,000 and each single-family home for $490,000.

   a. How much profit does he make by selling each type of house?

   b. Write a function to represent the profit the developer makes for selling a certain number of each type of home.

   c. If the developer uses the plan worked out in Question 3 to minimize his costs, will he maximize his profit? Explain your reasoning.

Be prepared to share your solutions and methods.